

Extended BRS symmetry in topological field theories

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Abstract

A class of topological field theories like the BF model and Chern-Simons theory, when quantized in the Landau gauge, enjoys the property of invariance under a vector supersymmetry, which is responsible for their finiteness. We introduce a new type of gauge fixing which makes these theories invariant under an extended BRS symmetry, containing a new type of field, the ghost of diffeomorphisms. The presence of such an extension is naturally related to the vector supersymmetry discussed before.

1 Introduction

The BF model like the Chern-Simons theory are classically a class of topological models of Schwarz type [1]. Their topological nature [2]-[3] is ensured by the property of invariance not only under the BRS symmetry but also under a vector supersymmetry [4] making these models finite [5]-[6]-[7].

Such way of thinking doesn't take into account all the classical symmetries enjoyed by these models, like the invariance under diffeomorphisms, that the BRS quantization breaks in an explicit way. We have searched for another type (non-standard) of BRS quantization which can take into account the classical diffeomorphisms. We are going to show in this letter that it is possible to quantize the classical model with a new type of gauge fixing such that the quantum action is invariant under an extended BRS symmetry, obtained as a sum of the standard piece due to gauge invariance and another one due to the diffeomorphisms.

2 The classical BF model

The action of the BF model in two dimensions is given by

$$S_{inv}(A, \phi) = -\frac{1}{2}Tr \int d^2x \epsilon^{\mu\nu} F_{\mu\nu} \phi \quad (2.1)$$

where $\epsilon^{\mu\nu}$ is the completely antisymmetric tensor, $F_{\mu\nu} = F_{\mu\nu}^a T^a = \partial_\mu A_\nu - \partial_\nu A_\mu - i[A_\mu, A_\nu]$ is the field strength $A_\mu(x) = A_\mu^a(x)T^a$ and $\phi(x) = \phi^a(x)T^a$ are respectively the gauge field and the scalar field, both belonging to the adjoint representation of the gauge group.

The action (2.1) is classically invariant :

-i) under the gauge transformations, whose infinitesimal parameter is $\Lambda(x) = \Lambda^a(x)T^a$

$$\begin{aligned} \delta_g A_\mu &= D_\mu \Lambda = \partial_\mu \Lambda - i[A_\mu, \Lambda] \\ \delta_g \phi &= -i[\phi, \Lambda] \end{aligned} \quad (2.2)$$

-ii) under the classical diffeomorphisms with $\xi^\mu(x)$ as an infinitesimal parameter

$$\begin{aligned} \delta_\xi A_\mu &= \xi^\nu \partial_\nu A_\mu + (\partial_\mu \xi^\nu) A_\nu \\ \delta_g \phi &= \xi^\nu \partial_\nu \phi \end{aligned} \quad (2.3)$$

Usually the quantization of the model is obtained with the introduction of quantum fields ($c^a(x), \bar{c}^a(x), b^a(x)$) playing the role respectively of ghost, antighost and Lagrange multiplier.

In terms of those fields the gauge fixing reads in the Landau gauge:

$$S_{gh} = \int d^2x \left(b^a \partial^\mu A_\mu^a - \bar{c}^a \partial^\mu (D_\mu c)^a \right) \quad (2.4)$$

The usual gauge transformations are no more a symmetry of the gauge fixed quantum action $S = S_{inv}(A, \phi) + S_{gf}(A, c, \bar{c}, b)$, which is however invariant under the *BRS* transformations:

$$\begin{aligned} s_c A_\mu &= D_\mu c \\ s_c \phi &= -i[\phi, c] \\ s_c c &= ic^2 \\ s_c \bar{c} &= b \\ s_c b &= 0 \end{aligned} \quad (2.5)$$

The *BRS* operator s_c is characterized by two fundamental properties:

- i) being nilpotent $s_c^2 = 0$
- ii) being a symmetry of the quantum action

$$s_c S = s_c (S_{inv} + S_{gf}) = 0 \quad (2.6)$$

The last property (2.6) is easily verified once that it is noticed that the gauge fixing S_{gf} is s_c -exact:

$$S_{gf}(b, c, \bar{c}, A) = s_c \int d^2x \bar{c}^a \partial^\mu A_\mu^a \quad (2.7)$$

The topological action S_{inv} doesn't depend from the metric of the space-time $g_{\mu\nu}$. In other words, only the gauge fixing term of the action gives contribution to the energy-momentum tensor, which is therefore an exact *BRS* cocycle:

$$T_{\mu\nu} = \frac{\partial S}{\partial g^{\mu\nu}} = s_c \Lambda_{\mu\nu} \quad (2.8)$$

The non trivial observation that both $T_{\mu\nu}$ and $\Lambda_{\mu\nu}$ are conserved is the signal of an additional symmetry of the action. The conservation law

$$\partial^\nu \Lambda_{\mu\nu} = \text{contact terms} \quad (2.9)$$

once integrated represents directly the Ward identity of the vector supersymmetry:

$$\begin{aligned} \delta_\mu A_\nu &= 0 \\ \delta_\mu \phi &= \epsilon_{\mu\nu} \partial^\nu \bar{c} \\ \delta_\mu c &= -A_\mu \\ \delta_\mu \bar{c} &= 0 \\ \delta_\mu b &= \partial_\mu \bar{c} \end{aligned} \quad (2.10)$$

The existence of such linear vector symmetry is peculiar to the topological field theories. It satisfies the following on-shell algebra:

$$\begin{aligned} s_c^2 &= 0 \\ \{\delta_\mu, \delta_\nu\} &= 0 \\ \{\delta_\mu, s_c\} &= \partial_\mu \end{aligned} \quad (2.11)$$

Our point of view is observing that the classical action (2.1) is invariant under infinitesimal diffeomorphisms and that such invariance is ruined by the presence of the gauge fixing.

We have searched to modify the gauge fixing in order to incorporate such an invariance at a quantum level. The answer turned out to be positive introducing an additional *BRS* operator for the diffeomorphisms [8] :

$$\begin{aligned} s_\xi \xi^\mu &= \xi^\nu \partial_\nu \xi^\mu \\ s_\xi \phi &= \xi^\nu \partial_\nu \phi \\ s_\xi A_\mu &= \xi^\nu \partial_\nu A_\mu + (\partial_\mu \xi^\nu) A_\nu \end{aligned} \quad (2.12)$$

with $\xi^\mu(x)$ a new field which we call ghost for the diffeomorphisms, that shall be treated as an anticommuting variable. Such *BRS* operator s_ξ enjoys the property of nilpotency:

$$s_\xi^2 = 0 \quad \leftrightarrow \quad \{\xi^\mu, \xi^\nu\} = 0 \quad (2.13)$$

It remains to redefine the quantum action

$$\tilde{S} = S_{inv}(A, \phi) + \tilde{S}_{gf}(A, b, c, \bar{c}, \xi) \quad (2.14)$$

in order that it is totally *BRS* invariant. In order to reach such aim we define the new gauge fixing as :

$$\begin{aligned} \tilde{S}_{gf}(A, b, c, \bar{c}, \xi) &= s \int d^2x \bar{c}^a \partial^\mu A_\mu^a \\ s &= s_c + s_\xi \end{aligned} \quad (2.15)$$

where $s = s_c + s_\xi$ is the total *BRS* operator. The modified action (2.15) is still invariant under the vector supersymmetry (2.10).

Having defined the action of s_ξ under the gauge ghost as:

$$\begin{aligned} s_\xi \bar{c} &= \xi^\mu \partial_\mu \bar{c} \\ s_\xi A_\mu &= \xi^\nu \partial_\nu A_\mu + (\partial_\mu \xi^\nu) A_\nu \end{aligned} \quad (2.16)$$

it is not difficult to isolate the contribution to the quantum action due to the ghost of diffeomorphisms

$$\begin{aligned} S_\xi &= \int d^2x \xi^\nu [\partial_\nu \bar{c} \partial^\mu A_\mu + \partial_\mu \bar{c} (\partial_\mu A_\nu - \partial_\nu A_\mu) + \partial_\mu \partial_\mu \bar{c} A_\nu] = \\ &= \int d^2x \xi^\nu \delta_\nu L(A, \phi, b, c, \bar{c}) \quad S = \int d^2x L \end{aligned} \quad (2.17)$$

We believe that it is not a coincidence that this contribution is proportional to the variation of the Lagrangian under the vector supersymmetry already met before (eq. (2.10)), with the ghost of diffeomorphisms playing the role of Lagrange multiplier.

3 Chern-Simons action

The Chern-Simons theory, once quantized in the usual manner in the Landau gauge, leads to the following action [9] :

$$S = S_{inv}(A) + S_{gf}(A, c, \bar{c}, b) = \int d^3x \text{Tr} \left[-\frac{1}{2}\epsilon^{\mu\nu\rho}(A_\mu\partial_\nu A_\rho + \frac{1}{3}A_\mu[A_\nu, A_\rho]) + b\partial_\mu A^\mu - \bar{c}\partial^\mu D_\mu c \right] \quad (3.1)$$

and the associated *BRS* transformations take the following form

$$\begin{aligned} s_c^1 A_\mu &= D_\mu c \\ s_c^1 c &= c^2 \\ s_c^1 \bar{c} &= b \\ s_c^1 b &= 0 \end{aligned} \quad (3.2)$$

The topological nature of this theory is ensured by an additional vector supersymmetry

$$\begin{aligned} \delta_\rho^1 A_\mu &= \epsilon_{\mu\rho\nu}\partial^\nu c \\ \delta_\rho^1 c &= 0 \\ \delta_\rho^1 \bar{c} &= A_\rho \\ \delta_\rho^1 b &= D_\rho c \end{aligned} \quad (3.3)$$

It has been noticed in [9] that this theory admits another anti-*BRS* symmetry, obtained by replacing the fields in this way

$$A_\mu \rightarrow A_\mu, \quad c \rightarrow \bar{c}, \quad \bar{c} \rightarrow -c, \quad b \rightarrow b - \{c, \bar{c}\} \quad (3.4)$$

and this extends the *BRS* – *SUSY* symmetry to an $N = 2$ superalgebra. By indicating the anti-*BRS* and anti-*SUSY* transformations with s_c^2 and δ_μ^2 , the on-shell algebra takes the form:

$$\{\delta_\mu^i, \delta_\nu^j\} = \epsilon^{ij}\epsilon_{\mu\nu\tau}\partial^\tau \quad \{s_c^i, \delta_\mu^j\} = \epsilon^{ij}\partial_\mu \quad (3.5)$$

Again our trick works by adding to the standard gauge-fixing of the Landau gauge the contribution of the ghost of diffeomorphisms:

$$\begin{aligned} \tilde{S}_{gf}(A, b, c, \bar{c}, \xi) &= s \int d^3x \bar{c}^a \partial^\mu A_\mu^a \\ s &= s_c^1 + s_\xi \end{aligned} \quad (3.6)$$

The part of the action containing the ghost of the diffeomorphisms is again strictly related to the vector supersymmetry of type δ^2 :

$$S_\xi = \int d^3x \, \xi^\mu \delta_\mu^2 L(A, b, c, \bar{c}) \quad S = \int d^3x \, L \quad (3.7)$$

4 Conclusion

We have seen in this letter that topological field theories in the Landau gauge enjoy many interesting properties. We have found a new choice of the gauge fixing such that the quantum model is invariant under an extended BRS symmetry including the vector supersymmetry in a natural way. We hope that our suggestion may help in elaborating further developments of the quantum properties of these models.

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